XIV. *On the Resistance of the Air to the Motion of Elongated Projectiles having variously formed Heads.*

By F. BASHFORTH, *B.D., Professor of Applied Mathematics to the Advanced Class of Artillery Officers, Woolwich, and late Fellow of St. John's College, Cambridge. Communicated by Professor* STOKES, *Sec. R.S.*

Received January 30,—Read February 20, 1868.

THE famous theory of the parabolic motion of projectiles was at an early period found to give results not in accordance with practice. Manifestly, then, the air must offer a very sensible resistance to a body which is moving through it with a high velocity. This resistance will depend upon the *form* of the moving body, and upon the *velocity* with which it is moving. Hence, before the path of a projectile can be calculated, it will be necessary to determine experimentally the resistance opposed by the air to the motion of the projectile, corresponding to various velocities. According to NEWTON'S law, the resistance of the air varies as the square of the velocity. But the velocities were low in the experiments made under his direction. In 1719 JOHN BERNOULLI gave equations for finding by the method of Quadratures the path &c. of a projectile, when the resistance of the air was supposed to vary according to any power of the velocity. But in spite of grave doubts respecting the accuracy of NEWTON'S law, it has been adopted by most of the eminent mathematicians who have written on the subject, such as EULER (1753), LAMBERT (1765), BORDA (1769), BEZOUT (1789), TEMPELHOF (1788–9), D'EHRENMALM (1788), LOMBARD (1796), and POISSON.

The first good experiments made with a view to determine the resistance of the air to the motion of projectiles were those of ROBINS in 1742. The projectiles used were leaden bullets of small size. When we consider the great density of the material used, its liability to change its form in the barrel of the gun, and the smallness of the *solid* projectiles, it is truly wonderful that ROBINS was able to accomplish so much with his ballistic pendulum. Afterwards HUTTON carried on ROBINS' system of experimenting both with the whirling machine and ballistic pendulum, introducing additional precautions, and using iron projectiles of greater size. In recent times MM. DIDION, MORIN, and PIOBERT have carried on experiments in France with heavier spherical projectiles, by the help of an improved ballistic pendulum; but they have done little more than confirm the results of ROBINS and HUTTON, and extend them to spherical projectiles of larger diameter.

ROBINS came to the conclusions:—“ *First*, That, till the velocity of the projectile surpasses that of 1100 feet in a second, the resistance may be esteemed to be in the duplicate proportion of the velocity; and its mean quantity may be taken to be nearly the

same with that I have assigned in the former paper. *Second*, That, if the velocity be greater than that of 1100 or 1200 feet in a second, then the absolute quantity of that resistance in these greater velocities will be near three times as great as it should be by a comparison with the smaller velocities"*. HUTTON remarks in a note on these conclusions:—" These suppositions are not nearly correct. In fact, by more accurate experiments with cannon-balls, it appears that the law of the resistance begins to increase above the ratio of the square of the velocity, from the very slowest motions, and thence goes on increasing gradually more and more above what is assigned by that ratio, till we arrive at the velocity of 1600 or 1700 feet per second, where it is at the greatest, amounting in that maximum state to only $2\frac{1}{10}$ times the quantity resulting from the ratio of the square of the velocity. And at the velocity of 1100 feet, instead of answering to that law, it amounts to 1·86 times the same." EULER, in the remarks which accompany his translation of ROBINS' 'Gunnery,' states that, the greater the velocity of the shot, so much the more does theory deviate from the truth†. HUTTON's formula of resistance consisted of two terms, one varying as the velocity, and the other as the square of the velocity.

In the year 1836 M. PIOBERT reexamined HUTTON's experiments, and found that the resistance of the air for various velocities was sufficiently well represented by a formula of two terms, one of which varied as the square, and the other as the cube of the velocity. In 1839 and 1840 numerous experiments were made at Metz, under the direction of a commission, by means of an improved ballistic pendulum. The projectiles used were spherical solid shot of 24, 12, and 8, or 26·47 lbs., 13·38 lbs., and 8·86 lbs. in weight, and 5·85 inches, 4·66 inches, and 4·06 inches in diameter, and a shell 50·71 lbs. in weight and 8·67 inches in diameter. The distances from the gun at which the pendulum was placed were 49 feet, 82 feet, 164 feet, 246 feet, and 328 feet. The resistance of the air to these projectiles was found to be represented by the formula

$$\pi R^2 v^2 \times 0{\cdot}027(1+0{\cdot}0023v)\ddagger;$$

and the new calculation of HUTTON's experiments gave

$$\pi R^2 v^2 \times 0{\cdot}02786(1+0{\cdot}0023v)\ddagger.$$

When spherical balls and smooth-bored guns were used, it was only possible to strike the receiver properly when at a moderate distance from the gun; and thus the variation of velocity to be measured was confined within very narrow limits. There was also the disadvantage that, as the velocity of the ball had to be reduced to that of the receiver in order to determine the striking velocity of the ball, only one velocity could be measured for each round fired. It would therefore be quite impossible to employ ROBINS' ballistic pendulum to find the velocities of the heavy elongated projectiles in use at the present day.

* ROBINS' Tracts on Gunnery, by HUTTON, 1805, p. 181.

† Neue Grundsätze der Artillerie, 1745, p. 508.

‡ DIDION, Traité de Balistique, 1860, pp. 61 & 64.

Various attempts have been made to measure the velocities of cannon-balls by the aid of electricity. The machines with revolving cylinders were in general failures, because their inventors made their success depend upon the known uniform angular velocity of the cylinder. These failures opened the way for the introduction of Major NAVEZ' electro-ballistic pendulum, and others of the same class, which worked with two screens, and therefore furnished no means for testing the probable accuracy of the velocity determined. The apparent convenience and portability of these instruments led to their general use both in Europe and America. Major NAVEZ' instrument, in its original complicated form, is now out of fashion; whilst Colonel BENTON'S two-pendulum instrument and Colonel LEURS' modification of it are in common use, as they are simpler than Major NAVEZ' instrument, and give results quite as much to be relied upon. Even if the electro-ballistic pendulum were perfect in itself as a measurer of time, considerable errors might be expected to arise from the imperfections of the indications of two screens placed at moderate distances apart. A reference to a paper by Major NAVEZ, "Considérations sur les expériences de balistique en ce qui concerne la mesure du temps"*, will show how little had been accomplished when that was written (1865). And Colonel BENET has well remarked:—"Electro-ballistic machines heretofore used have been powerless to solve one of the most important problems in ballistics—the law of the movement of a projectile through the air,—and this because of the limited number of points of the trajectory that could be determined"†. From preliminary experiments already made, I feel certain that a simpler, cheaper, and better instrument might be substituted with advantage for electro-ballistic pendulums where such instruments can be used. The time occupied by a body in falling from rest through a given space, or the time occupied by the sound of the explosion in travelling over a given space, might be made the foundation of the measurement of a velocity; or the velocity of the shot might be directly compared with the velocity of the sound of the explosion.

In the spring of 1864, when I was appointed Professor of Applied Mathematics to the Advanced Class of Artillery Officers at Woolwich, and Referee of the Ordnance Select Committee, I strongly recommended the construction of a chronograph capable of recording the time occupied by a projectile in passing over *nine* or more *successive equal spaces.* The principle of the chronograph used at the Greenwich Observatory was plainly the one to be adopted. The chief difficulties to be overcome were found (1) in the arrangement of a proper system of screens, so that the ball in passing might merely cause a *momentary interruption* (not a rupture) of the galvanic current, and that the resistance of the circuit might be kept *perfectly constant* during the experiment, (2) in the arrangement of a system of marking, which should give definite records on the surface of the cylinder when moving with a velocity of about 10 inches per second, and (3) in the compensation for the want of uniformity in the angular velocity of the cylinder.

* Revue de Technologie Militaire, t. iv.

† Electro-ballistic Machines, 1866, p. 39.

The chronograph was completed in the summer of 1865, and in November and December of the same year it was tried with ten screens, placed at intervals of 120 feet. Satisfactory records were obtained for eleven out of eighteen rounds of elongated shot fired from a 12-pounder B.L. gun, which very plainly indicated that the resistance of the air varied as the cube of the velocity. A full account of the chronograph and of the method of using it, accompanied by a detailed statement of the eleven successful rounds, was printed in the Proceedings of the Royal Artillery Institution, Woolwich, for August 1866, and was also published in a separate form. At present it will be sufficient to state that the axis of the cylinder is vertical, and in a line with the axis of the horizontal fly-wheel, to which it is attached. The fly-wheel is spun by hand. When the gun is ready to be fired, the markers are brought into contact with the paper; and after the clock has recorded three or four seconds the gun is fired. The clock is allowed to record three or four seconds more, and then the markers are raised and the experiment is completed. The friction on the axis of the fly-wheel, the resistance of the air to the motion of the wheel, and the friction of the markers on the paper tend to reduce the angular velocity of the cylinder. But as the pendulum of a half-seconds' clock raises a lever once each double swing, it interrupts the clock galvanic current once a second, and thus the clock-records show what spaces have passed under the markers each second. The changing angular velocity of the cylinder thus becomes accurately known. For if we commence measuring from some arbitrary point taken two or three seconds before the screen-records, and measure along the spiral traced by the clock marker, noting each record of the clock, and continuing our measurements two or three seconds beyond the screen-records, and if we difference these quantities, we shall find whether the angular velocity has been sufficiently regular in its change during the experiment. If so, we can by interpolation find what would have been the records if one had been made every *tenth* of a second. It has always been found to be sufficient to suppose the angular velocity constant during each tenth of a second, and to calculate smaller intervals of time by proportional parts.

As the clock goes on all day breaking the current once a second, every record of the clock is made under precisely the same circumstances. If there be a loss of time between the breaking of the current and the making of the corresponding record, the loss of time may always be expected to remain the same for any single experiment, and therefore there can be no error; for in experiments on gunnery the exact length of a second, only, is required, and not the exact time of the day. The galvanic current which works the screen-marker is kept constantly circulating through all the screens, excepting during the momentary interruption caused by the breaking or repairing of a screen, or some accidental rupture of the conducting wire. Although there is no necessity for this arrangement, it is found to be practically convenient. The ordinary screens used for other instruments are formed of fine copper wire stretched across a frame repeatedly, and through which the galvanic current circulates. When, then, a shot passes through the screen the current is permanently broken, unless some of the broken wires happen

to rest in contact. But for my chronograph it was necessary to make provision for the breaking and immediate restoration of the current, and that without changing the resistance of the circuit. The galvanic current passes along the top of each of the ten screens simultaneously. Equal weights are attached by long pieces of sewing-cotton to certain wire springs which project through holes in sheet copper. When the shot cuts one or more threads, the corresponding springs are released and fly from the bottom to the top of their holes. So long as any single spring is not in contact with the side of the hole through which it projects, the current is interrupted. This kind of screen secures a perfectly constant resistance to the screen galvanic current. But it has been objected that if the galvanic current circulated about the screen electro-magnet for several minutes, which elapse between successive rounds, the loss of time between the breaking of the galvanic current and its corresponding registration would *not be equal* for all the ten screens. This error would be the more pernicious because it might be expected to *follow some law*, and therefore could not be eliminated. The following arrangements have been made partly to meet this difficulty, and partly for the sake of the convenience of keeping up a constant communication between the instrument and the range.

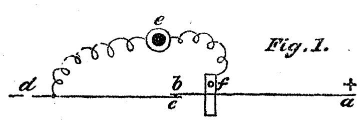


Fig. 1.

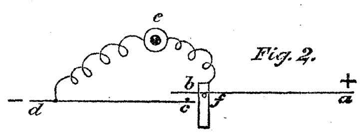
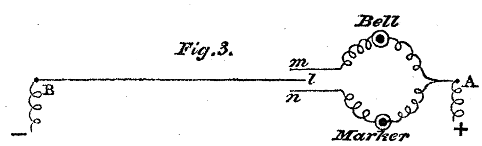


Fig. 2.

A self-acting contact-breaker and ringer (figs. 1 & 2) is placed by the side of the gun. Ordinarily the lever *a b* is down, as in fig. 1, in which case the galvanic current takes the direct course, *a b c d*. When the lever *a b* is raised, the current is permanently interrupted, but the insertion of a metallic pin *f* (fig. 2) opens a passage through the contact-breaker, *e*, when all the bells in the circuit are continuously rung. After the screens have been mended the lever *a b* is raised to try whether the current is good. If so, it is lowered and the gun is loaded. When the range is clear and all is ready, the lever *a b* is raised and the pin *f* is inserted. The fly-wheel of the chronograph is now put in motion. The rapid interruptions of the current by the contact-breaker are recorded on the cylinder, till a pull at the lanyard, to fire the gun, simultaneously withdraws also the pin *f*, and so shuts out the contact-breaker. The screen-records often follow so closely that it is difficult, before measurement, to say which is the first screen-record.

Fig. 3.



Another improvement, represented in fig. 3, has recently been introduced. The two ends of the circuit are at A and B, near the chronograph. When the markers are being raised from the paper by a lever, the spring *l* is simultaneously brought into contact with *m*, which turns the screen-current from the marker to the bell. Thus every interruption of the screen-current caused by the repair of the screens is signaled by the bell. When the bell rings continuously, it is known that the lever *a b* (fig. 2) is raised ready for firing. The fly-wheel is caused to spin, the markers are brought down upon the paper, and *simultaneously* the spring *l* is brought into contact with *n*, when the bell is silenced and the

marker registers the breaks of the contact-breaker till the pin f is withdrawn and the gun is fired. The clock is allowed to make a few beats; and then the markers are raised from the paper, and contact is reestablished between l and m. Thus the galvanic current only circulates for eight or ten seconds about the screen electro-magnet for each experiment, and the current is always being rapidly interrupted quite up to the firing of the gun; so that there is no opportunity for the development of a varying strength of remaining magnetism. The whole arrangement is found to work so satisfactorily that on one occasion nine rounds (23 to 31) were fired in forty-five minutes.

After all possible precautions have been taken, it is found that there are small corrections required in order to make the successive records of both clock and screen consistently regular. The unit of the scale used in measuring is about half an inch, and the scale is read off to two places of decimals, or to the $\frac{1}{200}$ of an inch. The corrections are carried to three places of decimals of the scale. The final calculations are carried to four places of decimals of a second. This is done to secure accuracy to the nearest $\frac{1}{1000}$ of a second of time, giving an opening for an error of $\pm\frac{1}{2000}$ of a second of time, or 6 or 8 inches of space, in finding the time occupied by the shot in passing from the first to any succeeding screen. These corrections of the readings of the scale are rendered necessary because the screens cannot be practically maintained at perfectly equal distances. The point of the shot may strike fairly upon a thread at one screen, and between two threads at the next screen. One spring may act more promptly than another. One string may bend more than another before breaking. These corrections are often merely nominal, but there are some sufficiently large to warn us to beware of trusting implicitly to any measurement of a velocity by two screens only.

Shortly after the publication of the description of my chronograph, my attention was directed to a chronograph with a cylinder, the invention of Captain SCHULTZ*, of the French Artillery, which had been tried in France and America. The instrument is adapted for making any number of records; and, like my own, its success does not depend upon the uniformity of rotation of the cylinder. My instrument makes the clock- and screen-records side by side, on glazed paper which covers the cylinder, so that the original records of the experiments can be preserved for future reference. Captain SCHULTZ makes his records on the slightly smoked metal surface of his cylinder, which are effaced when they have been read off. Captain SCHULTZ uses a large tuning-fork, usually called a diapason, the vibrations of which are sustained by electro-magnetism, to effect the mechanical division of the second into 250 or more equal intervals. The diapason, vibrating as the cylinder turns, traces a sinuous spiral line. The pendulum in swinging interrupts a galvanic circuit once a second, and causes a spark from a Ruhmkorff's coil to strike the cylinder and make a record. Thus it is found how many vibrations the diapason makes per second. The clock is then taken out of the circuit, and the current is made to pass through the Ruhmkorff coil and the first screen. When the first screen is broken the coil gives a spark, and the galvanic current is made to pass through the

* Colonel BENET's 'Electro-ballistic Machines,' 1866, p. 32.

second screen*. When the second screen is broken, a recording spark is given by the coil, and the current is passed on to the third screen, and so on to the end. All the time, the diapason is tracing its sinuous spiral. Unless a careful system of compensation be provided, this method of working the screens would cause great variation in the resistance of the circuit. In the arrangement of my screens, I was careful to maintain a *constant resistance* to the current, which end is secured by making the current pass through all the screens simultaneously, and by providing for it to be interrupted, but not broken. I am not aware that Captain SCHULTZ' instrument has been tried with more than two screens; but Colonel BENET has given a Table showing the number of vibrations per second made by the diapason as determined at the Frankfort Arsenal. In this Table there are striking variations in the numbers of vibrations made in successive seconds, as in the second trial, extending to twenty-five seconds, we find 249·1, 252·0, 249·5, 248·5, 246·0, 249·0, &c. The result of the trials is stated as if the constancy of the *mean* number of vibrations per second was all that was required. This is not the case. A succession of *equal* intervals of time must be marked out by the clock, or diapason, so that, when the gun is fired, the time of passing the screens may be noted by the side of a *correct* scale of time. Now it would make an important difference in the resulting velocity, if the gun were fired when the diapason was making 252·0, 248·5, or 246·0 vibrations per second. If the vibrations of the diapason be maintained by the vibrations of a second fork which alternately makes and breaks contact, as described by HELMHOLTZ†, it is hardly to be expected that the number of vibrations per second can be maintained with sufficient constancy. There is another question. The point from which the spark is discharged cannot be allowed to touch the smoked surface of the cylinder; and it may be asked whether the spark is not liable to deviate in its passage.

After my chronograph had passed its first trial (in December 1865), it appeared to be desirable to institute experiments with a view to find the resistance of the air to various forms of heads of elongated shot, but such as were likely to be of practical utility.

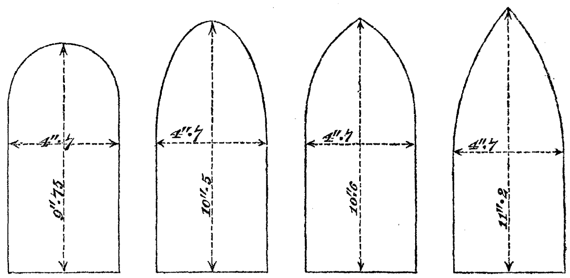
The proposed experiments received the sanction of the Right Honourable the Secretary of State for War, and ten of each kind of the following elongated shot were prepared for the 40-pounder M.L. gun:—

(1) Solid. Hemispherical head.
(2) Solid. Hemispheroidal head (ratio of diameters 1:2).
(3) Solid. Ogival, struck with radius equal to a diameter of shot.
(4) Solid. Ogival, struck with radius equal to two diameters of shot.
(5) Hollow. Ogival, having precisely the same external form and length as (3).
(6) Hollow. Ogival, having precisely the same external form and length as (4).

* "In working the instrument it is essential that the current pass only through one target at a time, there being but one coil and one battery no matter how many targets may be used. After the first target is ruptured, the current must be transferred to the succeeding one before the projectile reaches it, and so on throughout the series."—BENET, p. 36. See also the Practical Mechanic's Journal, Oct. 1, 1867, p. 195, to the same effect.

† Tonempfindung, p. 584.

Forms of Shot.



(1) Hemispherical head. (2) Hemispheroidal head. (3) and (5) Ogival head (1 diam.). (4) and (6) Ogival head (2 diams.).

Rounds 1–13 were fired on the 25th September 1866.
14–31 „ „ „ 26th September 1866.
32–43 „ „ „ 27th September 1866.

A barge having anchored in the range, there was no opportunity of firing a single round during the whole of the following day, which interfered with the completeness of the experiment with the solid shot. As a uniform charge of 5 lbs. of powder was used for all the rounds fired, and as the solid were nearly double the weight of the hollow shot, the hollow shot had a much higher initial velocity than the solid. Hence the hollow shot had also a much higher initial angular velocity than the solid shot; and it is probable that the initial angular velocity would be preserved nearly unimpaired throughout the observed range. This is manifested in the greater steadiness of the hollow ogivals.

For the first round, the method of forming the Time-table, and the mode of using it in the calculation of the times of arrival of the shot at each successive screen, are indicated at full length. For further details I must refer the reader to the published description of the chronograph.

The screens were placed 150 feet apart, which distance is denoted by l. The first screen was 75 feet from the gun. t denotes the time occupied by the shot in passing from the first screen to a distance s feet, when $t=as+bs^2$; and $0, t_2, t_3, \ldots t_n$ will denote the time when the shot passes the first, second, third, ... nth screen, or the times corresponding to the particular values $0, l, 2l, \ldots \overline{n-1}\, l$ of s.

(1) Hemispherical-headed Shot. Diameter 4·7 inches.

Round 1. Weight of shot 39·34 lbs.

Clock.

	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.
″1	4·91	0	4·910	+15·910	
2	20·82	0	20·820	15·840	−70
3	36·65	+·010	36·660	15·773	67
4	52·44	−·007	52·433	15·709	64
5	68·14	+·002	68·142	+15·648	−61
6	83·79	0	83·790		

Time-table obtained by interpolation.

″2·5	28·748	+1·584
2·6	30·332	1·583
2·7	31·915	1·582
2·8	33·497	1·582
2·9	35·079	1·581
3·0	36·660	1·580
3·1	38·240	1·580
3·2	39·820	1·579
3·3	41·399	1·578
3·4	42·977	1·578
3·5	44·555	1·577
3·6	46·132	1·576
3·7	47·708	1·576
3·8	49·284	1·575
3·9	50·859	+1·574
4·0	52·433	

Interpolation.

		Δ_1.	Δ_2.
″2·0	20·820	+7·928	
2·5	28·748	7·912	−16
3·0	36·660	7·895	−17
3·5	44·555	+7·878	−17
4·0	52·433		

Screens.

No.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.
1	28·26	0	28·260	+2·002	
2	30·26	+·002	30·262	2·030	+28
3	32·30	−·008	32·292	2·058	28
4	34·35	0	34·350	2·086	28
5	36·43	+·006	36·436	2·114	28
6	38·54	+·010	38·550	2·141	27
7	40·76	*	40·691	2·167	26
8	42·86	−·002	42·858	2·193	26
9	45·06	−·009	45·051	+2·219	+26
10	47·26	+·010	47·270		

Having thus obtained the space described by the clock marker at the end of every tenth of a second, from 2″·5 up to 4″·0, we can calculate the times when the screens were passed as follows:—

		″ t.	Δ_1.	Δ_2.
Screen 1	28·260 passed at ″2·5 $-\frac{488}{1584}\times$·01 = ″2·4692	″0·0000	+1264	
„ 2	30·262 passed at 2·6 $-\frac{70}{1584}\times$·01 = 2·5956	·1264	1282	+18
„ 3	32·292 passed at 2·7 $+\frac{377}{1582}\times$·01 = 2·7238	·2546	1301	19
„ 4	34·350 passed at 2·8 $+\frac{853}{1582}\times$·01 = 2·8539	·3847	1319	18
„ 5	36·436 passed at 3·0 $-\frac{224}{1581}\times$·01 = 2·9858	·5166	1338	19
„ 6	38·550 passed at 3·1 $+\frac{310}{1580}\times$·01 = 3·1196	·6504	1356	18
„ 7	40·691 passed at 3·2 $+\frac{871}{1579}\times$·01 = 3·2552	·7860	1373	17
„ 8	42·858 passed at 3·4 $-\frac{119}{1578}\times$·01 = 3·3925	·9233	1390	17
„ 9	45·051 passed at 3·5 $+\frac{496}{1577}\times$·01 = 3·5315	1·0623	+1407	+17
„ 10	47·270 passed at 3·7 $-\frac{438}{1576}\times$·01 = 3·6722	1·2030		

Round 13. Weight of shot 39·33 lbs.

Clock.	Reading.	Correction.	Corrected reading.	Δ.	Screen.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.	Time of passing each screen. t.	Δ_1.	Δ_2.
″1	15·51	0	15·510	+24·050	1	64·69	0	64·690	+3·025	+36	0″·0000	+1266	+16
2	39·56	0	39·560	23·953	2	67·72	−·005	67·715	3·061	36	·1266	1282	16
3	63·52	−·007	63·513	23·857	3	70·77	+·006	70·776	3·097	34	·2548	1298	14
4	87·37	0	87·370	+23·760	4	73·87	+·003	73·873	3·131	35	·3846	1312	16
5	111·13	0	111·130		5	77·01	−·006	77·004	3·166	35	·5158	1328	15
					6	80·17	0	80·170	3·201	36	·6486	1343	16
					7	* *	*	83·371	3·237	37	·7829	1359	16
					8	86·61	−·002	86·608	3·274	+37	·9188	1375	+17
					9	89·89	−·008	89·882	+3·311		1·0563	+1392	
					10	93·19	+·003	93·193			1·1955		

Round 34. Weight of shot 39·34 lbs.

Clock.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.	Screen.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.	Time of passing each screen. t.	Δ_1.	Δ_2.
″4	89·69	0	89·690	+21·470	−70	1	104·33	0	104·330	+2·718	+33	0″·0000	+1267	+16
5	111·17	−·010	111·160	21·400	−70	2	107·05	−·002	107·048	2·751	33	·1267	1283	16
6	132·55	+·010	132·560	+21·330		3	109·80	−·001	109·799	2·784	32	·2550	1299	16
7	153·89	0	153·890			4	112·58	+·003	112·583	2·816	35	·3849	1315	16
						5	115·40	−·001	115·399	2·851	36	·5164	1331	17
						6	118·24	+·010	118·250	2·887	38	·6495	1348	19
						7	121·16	−·023	121·137	2·925	36	·7843	1367	17
						8	124·06	+·002	124·062	2·961	+36	·9210	1384	+18
						9	127·02	+·003	127·023	+2·997		1·0594	+1402	
						10	130·02	0	130·020			1·1996		

Round 43. Weight of shot 39·34 lbs.

Clock.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.	Screen.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.	Time of passing each screen. t.	Δ_1.	Δ_2.
″1	24·05	0	24·050	+21·455	−70	1	75·83	0	75·830	+2·685	+35	0″·0000	+1260	+16
2	45·51	−·005	45·505	21·385	70	2	78·50	+·015	78·515	2·720	35	·1260	1276	17
3	66·88	+·010	66·890	21·315	−70	3	81·24	−·005	81·235	2·755	35	·2536	1293	18
4	88·21	−·005	88·205	+21·245		4	83·99	0	83·990	2·790	37	·3829	1311	18
5	109·45	0	109·450			5	86·78	0	86·780	2·827	36	·5140	1329	17
						6	89·61	−·003	89·607	2·863	37	·6469	1346	18
						7	92·48	−·010	92·470	2·900	37	·7815	1364	18
						8	95·37	0	95·370	2·937	+36	·9179	1382	+18
						9	98·31	−·003	98·307	+2·973		1·0561	+1400	
						10	101·28	0	101·280			1·1961		

Summary (1). Hemispherical-headed Shot.

Time occupied by shot in passing from the first to each of the other screens.

No. of round.	Screen 1.	Screen 2.	Screen 3.	Screen 4.	Screen 5.	Screen 6.	Screen 7.	Screen 8.	Screen 9.	Screen 10.
1	0″·0000	0″·1264	0″·2546	0″·3847	0″·5166	0″·6504	0″·7860	0″·9233	1″·0623	1″·2030
13	0·0000	·1266	·2548	·3846	·5158	·6486	·7829	·9188	1·0563	1·1955
34	0·0000	·1267	·2550	·3849	·5164	·6495	·7843	·9210	1·0594	1·1996
43	0·0000	·1260	·2536	·3829	·5140	·6469	·7815	·9179	1·0561	1·1961

Velocities at the following distances from the gun.

No. of round.	150 ft.	300 ft.	450 ft.	600 ft.	750 ft.	900 ft.	1050 ft.	1200 ft.	1350 ft.
	f.s.	f.s.	f.s.	f.s.	f.s.	f.s.	f.s.	f.s.	f.s.
1	1187	1170	1153	1137	1121	1106	1092	1079	1066
13	1185	1170	1156	1143	1130	1117	1104	1091	1078
34	1184	1169	1155	1141	1127	1113	1097	1084	1070
43	1190	1175	1160	1144	1129	1114	1100	1085	1071

No. of round.	Weight of shot.	Value of bl^2.	Difference from mean.
	lbs.		
1	39·34	·00091	+·00007
13	39·33	·00078	−·00006
34	39·34	·00082	−·00002
43	39·34	·00085	+·00001
Means	39·34	·00084	·00004

(2) Hemispheroidal-headed Shot (Axes 1 : 2), Solid.

Round 2. Weight of shot 38·72 lbs.

Clock.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.	Screen.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.	Time of passing each screen. t.	Δ_1.	Δ_2.
″ 2	24·44	0	24·440	+23·750	−44	1	103·28	+·005	103·285	+2·997	+24	″ 0·0000	+1267	+11
3	48·19	0	48·190	23·706	39	2	106·28	+·002	106·282	3·021	29	·1267	1278	13
4	71·90	−·004	71·896	23·667	34	3	109·31	−·007	109·303	3·049	31	·2545	1291	12
5	95·56	+·003	95·563	23·633	−29	4	112·35	+·002	112·352	3·080	32	·3836	1303	15
6	119·20	−·004	119·196	+23·604		5	115·43	+·002	115·432	3·112	34	·5139	1318	14
7	142·80	0	142·800			6	118·55	−·006	118·544	3·146	+35	·6457	1332	+15
						7	121·70	−·010	121·690	+3·181		·7789	+1347	
						8	124·87	+·001	124·871			·9136		

Round 7. Weight of shot 38·69 lbs.

Clock.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.	Screen.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.	Time of passing each screen. t.	Δ_1.	Δ_2.
″ 3	60·24	0	60·240	+21·640	−80	1	82·59	+·003	82·593	+2·748	+28	″ 0·0000	+1273	+13
4	81·88	0	81·880	21·560	80	2	85·34	+·001	85·341	2·776	28	·1273	1286	14
5	103·45	−·010	103·440	21·480	−80	3	88·13	−·013	88·117	2·804	30	·2559	1300	14
6	124·91	+·010	124·920	+21·400		4	90·92	+·001	90·921	2·834	30	·3859	1314	15
7	146·32	0	146·320			5	93·75	+·005	93·755	2·864	27	·5173	1329	13
						6	96·62	−·001	96·619	2·891	28	·6502	1342	14
						7	99·51	0	99·510	2·919	+27	·7844	1356	+13
						8	102·43	−·001	102·429	+2·946		·9200	+1369	
						9	105·35	+·025	105·375			1·0569		

Round 35. Weight of shot 38·69 lbs.

Clock.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.	Screen.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.	Time of passing each screen. t.	Δ_1.	Δ_2.
″ 3	52·97	0	52·970	+18·230	−63	1	68·09	0	68·090	+2·309	+20	″ 0·0000	+1268	+12
4	71·20	0	71·200	18·167	−64	2	**	*	70·399	2·329	23	·1268	1280	13
5	89·37	−·003	89·367	+18·103		3	**	*	72·728	2·352	23	·2548	1293	13
6	107·47	0	107·470			4	**	*	75·080	2·375	23	·3841	1306	13
						5	**	*	77·455	2·398	23	·5147	1319	14
						6	79·85	+·003	79·853	2·421	24	·6466	1333	14
						7	82·29	−·016	82·274	2·445	24	·7799	1347	14
						8	84·72	−·001	84·719	2·469	+23	·9146	1361	+13
						9	87·19	−·002	87·188	+2·492		1·0507	+1374	
						10	89·68	0	89·680			1·1881		

Round 40. Weight of shot 38·69 lbs..

Clock.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.
″ 4	87·30	0	87·300	+19·750	−50
5	107·05	0	107·050	19·700	−50
6	126·75	0	126·750	+19·650	
7	146·40	0	146·400		

Screen.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.	Time of passing each screen. t.	Δ_1.	Δ_2.
1	113·53	0	113·530	+2·491	+26	″ 0·0000	+1264	+14
2	116·02	+·001	116·021	2·517	26	·1264	1278	13
3	118·55	−·012	118·538	2·543	26	·2542	1291	14
4	121·08	+·001	121·081	2·569	27	·3833	1305	14
5	123·65	0	123·650	2·596	27	·5138	1319	14
6	126·23	+·016	126·246	2·623	26	·6457	1333	14
7	128·87	−·001	128·869	2·649	27	·7790	1347	14
8	131·52	−·002	131·518	2·676	+26	·9137	1361	+14
9	134·19	+·004	134·194	+2·702		1·0498	+1375	
10	136·90	−·004	136·896			1·1873		

Summary (2). Hemispheroidal-headed Shot.

Time occupied by shot in passing from the first to each of the other screens.

No. of round.	Screen 1.	Screen 2.	Screen 3.	Screen 4.	Screen 5.	Screen 6.	Screen 7.	Screen 8.	Screen 9.	Screen 10.
2	″ 0·0000	″ 0·1267	″ 0·2545	″ 0·3836	″ 0·5139	″ 0·6457	″ 0·7789	″ 0·9136	″ **	″ **
7	0·0000	·1273	·2559	·3859	·5173	·6502	·7844	·9200	1·0569	**
35	0·0000	·1268	·2548	·3841	·5147	·6466	·7799	·9146	1·0507	1·1881
40	0·0000	·1264	·2542	·3833	·5138	·6457	·7790	·9137	1·0498	1·1873

Velocities at the following distances from the gun.

No. of round.	150 ft.	300 ft.	450 ft.	600 ft.	750 ft.	900 ft.	1050 ft.	1200 ft.	1350 ft.
	f.s.	*f.s.*	*f.s.*	*f.s.*	*f.s.*	*f.s.*	*f.s.*	*f.s.*	*f.s.*
2	1184	1174	1162	1151	1138	1126	1114	**	**
7	1178	1166	1154	1142	1129	1118	1106	1096	**
35	1183	1172	1160	1149	1137	1125	1114	1102	1092
40	1187	1174	1162	1149	1137	1125	1114	1102	1091

No. of round.	Weight of shot.	Value of bl^2.	Difference from mean value.
	lbs.		
2	38·72	·00064	−·00003
7	38·69	·00069	+·00002
35	38·69	·00065	−·00002
40	38·69	·00069	+·00002
Means	38·70	·00067	·00002

(3) Ogival-headed Shot (one diameter), Solid.

Round 3. Weight of shot 39·56 lbs.

Clock.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.
″ 1	21·09	0	21·090	+25·850	−100
2	46·94	0	46·940	25·750	79
3	72·69	0	72·690	25·671	62
4	98·36	+·001	98·361	25·609	− 49
5	123·97	0	123·970	+25·560	
6	149·53	0	149·530		

Screen.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.	Time of passing each screen. t.	Δ_1.	Δ_2.
1	114·12	0	114·120	+3·310	+35	″ 0·0000	+1293	+14
2	117·43	0	117·430	3·345	43	·1293	1307	18
3	120·78	−·005	120·775	3·388	48	·2600	1325	18
4	124·15	+·013	124·163	3·436	49	·3925	1343	20
5	127·60	−·001	127·599	3·485	46	·5268	1363	18
6	131·09	−·006	131·084	3·531	+39	·6631	1381	+16
7	134·62	−·005	134·615	+3·570		·8012	+1397	
8	138·18	+·005	138·185			·9409		

Round 36. Weight of shot 39·56 lbs.

Clock.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.	Screen.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.	Time of passing each screen. t.	Δ_1.	Δ_2.
″ 3	52·79	0	52·790	+19·700	−105	1	99·25	0	99·250	+2·466	+27	″ 0·0000	+1264	+15
4	72·49	0	72·490	19·595	100	2	101·29	*	101·716	2·493	26	·1264	1279	15
5	92·08	+·005	92·085	19·495	−95	3	104·21	−·001	104·209	2·519	25	·2543	1294	13
6	111·58	0	111·580	+19·400		4	106·73	−·002	106·728	2·544	25	·3837	1307	13
7	130·98	0	130·980			5	109·27	+·002	109·272	2·569	25	·5144	1320	15
						6	111·84	+·001	111·841	2·594	26	·6464	1335	14
						7	114·44	−·005	114·435	2·620	+25	·7799	1349	+13
						8	117·05	+·005	117·055	+2·645		·9148	+1362	
						9	119·70	0	119·700			1·0510		

Round 41. Weight of shot 39·56 lbs.

Clock.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.	Screen.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.	Time of passing each screen. t.	Δ_1.	Δ_2.
″ 2	53·16	−·006	53·154	+23·494	−90	1	123·37	0	123·370	+2·931	+31	″ 0·0000	+1260	+14
3	76·64	+·008	76·648	23·404	90	2	126·30	+·001	126·301	2·962	31	·1260	1274	14
4	100·06	−·008	100·052	23·314	−90	3	129·26	+·003	129·263	2·993	30	·2534	1288	13
5	123·36	+·006	123·366	+23·224		4	132·26	−·004	132·256	3·023	29	·3822	1301	13
6	146·59	0	146·590			5	135·27	+·009	135·279	3·052	29	·5123	1314	14
						6	138·33	+·001	138·331	3·081	29	·6437	1328	13
						7	141·41	+·002	141·412	3·110	29	·7765	1341	13
						8	144·51	+·012	144·522	3·139	+29	·9106	1354	+13
						9	147·65	+·011	147·661	+3·168		1·0460	+1367	
						10	150·84	−·011	150·829			1·1827		

Summary (3). Ogival-headed Shot (one diameter), Solid.

Time occupied by shot in passing from the first to each of the other screens.

No. of round.	Screen 1.	Screen 2.	Screen 3.	Screen 4.	Screen 5.	Screen 6.	Screen 7.	Screen 8.	Screen 9.	Screen 10.
3	″ 0·0000	″ 0·1293	″ 0·2600	″ 0·3925	″ 0·5268	″ 0·6631	″ 0·8012	″ 0·9409	″ *	″ *
36	0·0000	·1264	·2543	·3837	·5144	·6464	·7799	·9148	1·0510	*
41	0·0000	·1260	·2534	·3822	·5123	·6437	·7765	·9106	1·0460	1·1827

Velocities at the following distances from the gun.

No. of round.	150 ft.	300 ft.	450 ft.	600 ft.	750 ft.	900 ft.	1050 ft.	1200 ft.	1350 ft.
	f.s.	*f.s.*	*f.s.*	*f.s.*	*f.s.*	*f.s.*	*f.s.*	*f.s.*	*f.s.*
3	1160	1147	1132	1117	1101	1086	1074	*	*
36	1187	1173	1159	1148	1136	1124	1112	1101	*
41	1190	1177	1165	1153	1142	1130	1119	1108	1097

No. of round.	Weight of shot.	Values of bl^2.
	lbs.	
3	39·56	·00085
36	39·56	·00071 }
41	39·56	·00068 }
Mean...	39·56	Mean of 2 rounds ·00070

(4) Ogival-headed Shot (two diameters), Solid.

Round 4. Weight of shot 38·56 lbs.

Clock.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.	Screen.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.	Time of passing each screen. t.	Δ_1.	Δ_2.
″ 1	17·50	0	17·500	+23·115	−80	1	63·78	−·001	63·779	+2·910	+33	″ 0·0000	+1266	+14
2	40·62	−·005	40·615	23·035	67	2	66·67	+·019	66·689	2·943	33	·1266	1280	15
3	63·64	+·010	63·650	22·968	−56	3	69·63	+·002	69·632	2·976	32	·2546	1295	15
4	86·63	−·012	86·618	+22·912		4	72·61	−·002	72·608	3·008	32	·3841	1310	14
5	109·53	0	109·530			5	75·62	−·004	75·616	3·040	31	·5151	1324	13
						6	78·65	+·006	78·656	3·071	28	·6475	1337	14
						7	81·73	−·003	81·727	3·099	29	·7812	1351	13
						8	84·83	−·004	84·826	3·128	+28	·9163	1364	+12
						9	87·95	+·004	87·954	+3·156		1·0527	+1376	
						10	91·11	0	91·110			1·1903		

Round 37. Weight of shot 38·48 lbs.

Clock.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.	Screen.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.	Time of passing each screen. t.	Δ_1.	Δ_2.
″ 4	51·40	0	51·400	+16·640	−80	1	71·99	0	71·990	+2·113	+23	″ 0·0000	+1275	+15
5	68·04	0	68·040	16·560	−80	2	74·10	+·003	74·103	2·136	21	·1275	1290	13
6	84·60	0	84·600	+16·480		3	76·24	−·001	76·239	2·157	21	·2565	1303	13
7	101·08	0	101·080			4	78·40	−·004	78·396	2·178	+18	·3868	1316	+13
						5	80·57	+·004	80·574	+2·196		·5184	+1329	
						6	82·77	0	82·770			·6513		

Round 42. Weight of shot 38·47 lbs.

Clock.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.	Screen.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.	Time of passing each screen. t.	Δ_1.	Δ_2.
″ 4	69·77	0	69·770	+14·960	−80	1	81·34	−·004	81·336	+1·836	+24	″ 0·0000	+1230	+16
5	84·74	−·010	84·730	14·880	−80	2	83·17	+·002	83·172	1·860	23	·1230	1246	17
6	99·61	0	99·610	+14·800		3	85·03	+·002	85·032	1·883	23	·2476	1263	16
7	114·41	0	114·410			4	86·91	+·005	86·915	1·906	23	·3739	1279	16
						5	88·82	+·001	88·821	1·929	23	·5018	1295	17
						6	90·75	0	90·750	1·952	22	·6313	1312	15
						7	92·71	−·008	92·702	1·974	22	·7625	1327	16
						8	94·68	−·004	94·676	1·996	+22	·8952	1343	+16
						9	96·66	+·012	96·672	+2·018		1·0295	+1359	
						10	98·69	+ 0	98·690			1·1654		

Summary (4). Ogival-headed Shot (two diameters), Solid.

Time occupied by shot in passing from the first to each succeeding screen.

No. of round.	Screen 1.	Screen 2.	Screen 3.	Screen 4.	Screen 5.	Screen 6.	Screen 7.	Screen 8.	Screen 9.	Screen 10.
4	″ 0·0000	″ 0·1266	″ 0·2546	″ 0·3841	″ 0·5151	″ 0·6475	″ 0·7812	″ 0·9163	″ 1·0527	″ 1·1903
37	0·0000	·1275	·2565	·3868	·5184	·6513	*	*	*	*
42	0·0000	·1230	·2476	·3739	·5018	·6313	·7625	·8952	1·0295	1·1654

Velocities at the following distances from the gun.

No. of round.	150 ft.	300 ft.	450 ft.	600 ft.	750 ft.	900 ft.	1050 ft.	1200 ft.	1350 ft.
	f.s.	*f.s.*	*f.s.*	*f.s.*	*f.s.*	*f.s.*	*f.s.*	*f.s.*	*f.s.*
4	1185	1172	1158	1145	1133	1122	1110	1100	1090
37	1176	1163	1151	1140	1129	*	*	*	*
42	1220	1204	1188	1173	1158	1143	1130	1117	1104

No. of round.	Weight of shot.	Values of bl^2.
	lbs.	
4	38·56	·00071 }
37	38·48	·00069 }
42	38·47	·00081
Mean...	38·52	Mean of 2 rounds ·00070

(5) Ogival-headed Shot (one diameter), Hollow.

Round 14. Weight of shot 21·78 lbs.

Clock.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.	Screen.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.	Time of passing each screen. t.	Δ_1.	Δ_2.
″ 1	27·48	−·001	27·479	+21·079		1	96·37	+·002	96·372	+2·087		″ 0·0000	+1001	
2	48·56	−·002	48·558	20·999	−80	2	98·46	−·001	98·459	2·139	+52	·1001	1026	+25
3	69·55	+·007	69·557	20·921	78	3	100·61	−·012	100·598	2·191	52	·2027	1051	25
4	90·47	+·008	90·478	20·841	80	4	102·79	−·001	102·789	2·243	52	·3078	1077	26
5	111·33	−·011	111·319	+20·761	−80	5	105·02	+·012	105·032	2·295	52	·4155	1103	26
6	132·08	0	132·080			6	107·32	+·007	107·327	2·346	51	·5258	1127	24
						7	109·68	−·007	109·673	2·396	50	·6385	1152	25
						8	112·09	−·021	112·069	2·447	51	·7537	1176	24
						9	114·52	−·004	114·516	+2·498	+51	·8713	+1198	+22
						10	117·01	+·004	117·014			·9911		

Round 16. Weight of shot 21·81 lbs.

Clock.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.	Screen.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.	Time of passing each screen. t.	Δ_1.	Δ_2.
″ 1	5·14	0	5·140	+26·490		1	58·75	+·005	58·755	+2·603		″ 0·0000	+ 986	
2	31·63	0	31·630	26·440	−50	2	61·37	−·012	61·358	2·662	+59	·0986	1008	+22
3	58·07	0	58·070	26·378	62	3	64·02	0	64·020	2·724	62	·1994	1032	24
4	84·45	−·002	84·448	+26·302	−76	4	66·74	+·004	66·744	2·790	66	·3026	1058	26
5	110·75	0	110·750			5	69·52	+·014	69·534	2·859	69	·4084	1084	26
						6	72·39	+·003	72·393	2·927	68	·5168	1110	26
						7	75·33	−·010	75·320	2·995	68	·6278	1136	26
						8	78·33	−·015	78·315	3·063	68	·7414	1162	26
						9	81·37	+·008	81·378	+3·131	+68	·8576	+1188	+26
						10	84·50	+·009	84·509			·9764		

Round 18. Weight of shot 21·81 lbs.

Clock.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.	Screen.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.	Time of passing each screen. t.	Δ_1.	Δ_2.
″ 1	28·12	−·005	28·115	+21·864		1	101·10	−·002	101·098	+2·153		″ 0·0000	+ 993	
2	49·99	−·011	49·979	21·799	−65	2	103·23	+·021	103·251	2·208	+55	·0993	1018	+25
3	71·77	+·008	71·778	21·739	60	3	105·46	−·001	105·459	2·263	55	·2011	1044	26
4	93·51	+·007	93·517	21·684	55	4	107·73	−·008	107·722	2·317	54	·3055	1069	25
5	115·21	−·009	115·201	+21·634	−50	5	110·03	+·009	110·039	2·371	54	·4124	1094	25
6	136·84	−·005	136·835			6	112·41	0	112·410	2·425	54	·5218	1120	26
						7	114·86	−·025	114·835	2·479	54	·6338	1145	25
						8	117·32	−·006	117·314	2·533	54	·7483	1170	25
						9	119·85	−·003	119·847	+2·587	+54	·8653	+1195	+25
						10	122·43	+·004	122·434			·9848		

Round 20. Weight of shot 21·83 lbs.

Clock.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.	Screen.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.	Time of passing each screen. t.	Δ_1.	Δ_2.
″ 1	11·95	0	11·950	+21·790	−70	1	94·50	0	94·500	+2·135	+56	″ 0·0000	+ 990	+27
2	33·74	0	33·740	21·720	70	2	96·64	−·005	96·635	2·191	56	·0990	1017	26
3	55·45	+·010	55·460	21·650	70	3	98·82	+·006	98·826	2·247	54	·2007	1043	26
4	77·11	0	77·110	21·580	70	4	101·08	−·007	101·073	2·301	52	·3050	1069	24
5	98·68	+·010	98·690	21·510	−70	5	103·37	+·004	103·374	2·353	51	·4119	1093	24
6	120·20	0	120·200	+21·440		6	105·73	−·003	105·727	2·404	50	·5212	1117	24
7	141·65	−·010	141·640			7	108·15	−·019	108·131	2·454	50	·6329	1141	24
						8	110·59	−·005	110·585	2·504	+50	·7470	1165	+23
						9	113·08	+·009	113·089	+2·554		·8635	+1188	
						10	115·64	+·003	115·643			·9823		

Round 22. Weight of shot 21·81 lbs.

Clock.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.	Screen.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.	Time of passing each screen. t.	Δ_1.	Δ_2.
″ 3	66·77	0	66·770	+20·455	−72	1	97·44	0	97·440	+2·016	+47	″ 0·0000	+ 990	+22
4	87·23	−·005	87·225	20·383	−71	2	99·45	+·006	99·456	2·063	47	·0990	1012	24
5	107·60	+·008	107·608	+20·312		3	101·52	−·001	101·519	2·110	48	·2002	1036	24
6	127·92	0	127·920			4	103·63	−·001	103·629	2·158	49	·3038	1060	25
						5	105·78	+·007	105·787	2·207	49	·4098	1085	24
						6	107·99	+·004	107·994	2·256	49	·5183	1109	24
						7	110·26	−·010	110·250	2·305	50	·6292	1133	26
						8	112·55	+·005	112·555	2·355	+50	·7425	1159	+25
						9	114·90	+·010	114·910	+2·405		·8584	+1184	
						10	117·32	−·005	117·315			·9768		

Round 24. Weight of shot 21·83 lbs.

Clock.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.	Screen.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.	Time of passing each screen. t.	Δ_1.	Δ_2.
″ 2	36·69	0	36·690	+24·950	− 98	1	71·78	0	71·780	+2·474	+61	″ 0·0000	+ 995	+26
3	61·65	−·010	61·640	24·852	106	2	74·26	−·006	74·254	2·535	61	·0995	1021	24
4	86·49	+·002	86·492	24·746	−112	3	76·79	−·001	76·789	2·596	62	·2016	1045	26
5	111·24	−·002	111·238	+24·634		4	79·39	−·005	79·385	2·658	62	·3061	1071	25
6	135·87	+·002	135·872			5	82·04	+·003	82·043	2·720	62	·4132	1096	26
						6	84·75	+·013	84·763	2·782	63	·5228	1122	25
						7	87·56	−·015	87·545	2·845	63	·6350	1147	27
						8	90·39	0	90·390	2·908	+63	·7497	1174	+25
						9	93·30	−·002	93·298	+2·971		·8671	+1199	
						10	96·27	−·001	96·269			·9870		

Round 26. Weight of shot 21·81 lbs.

Clock.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.	Screen.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.	Time of passing each screen. t.	Δ_1.	Δ_2.
″ 3	38·40	+·001	38·401	+18·106	−69	1	78·16	−·005	78·155	+1·795	+45	″ 0·0000	+ 997	+27
4	56·51	−·003	56·507	18·037	62	2	79·94	+·010	79·950	1·840	45	·0997	1024	24
5	74·54	+·004	74·544	17·975	−54	3	81·78	+·010	81·790	1·885	44	·2021	1048	25
6	92·52	−·001	92·519	+17·921		4	83·68	−·005	83·675	1·929	43	·3069	1073	24
7	110·44	0	110·440			5	85·59	+·014	85·604	1·972	43	·4142	1097	26
						6	87·57	+·006	87·576	2·015	42	·5239	1123	23
						7	80·60	−·009	89·591	2·057	42	·6362	1146	23
						8	91·65	−·002	91·648	2·099	+42	·7508	1169	+24
						9	93·74	+·007	93·747	+2·141		·8677	+1193	
						10	95·89	−·002	95·888			·9870		

Round 28. Weight of shot 21·83 lbs.

Clock.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.	Screen.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.	Time of passing each screen. t.	Δ_1.	Δ_2.
″ 1	27·41	+·010	27·420	+20·505	−52	1	77·15	+·010	77·160	+2·020	+50	″ 0·0000	+991	+23
2	47·93	−·005	47·925	20·453	51	2	79·19	−·010	79·180	2·070	50	·0991	1014	25
3	68·37	+·008	68·378	20·402	−52	3	81·25	0	81·250	2·120	50	·2005	1039	26
4	88·78	0	88·780	+20·350		4	83·37	0	83·370	2·170	50	·3044	1065	24
5	109·13	0	109·130			5	85·54	0	85·540	2·220	50	·4169	1089	25
						6	85·76	0	87·760	2·270	50	·5198	1114	26
						7	90·05	−·020	90·030	2·320	50	·6312	1140	24
						8	92·35	0	92·350	2·370	+50	·7452	1164	+24
						9	94·72	0	94·720	+2·420		·8616	+1188	
						10	97·14	0	97·140			·9804		

Round 30. Weight of shot 21·81 lbs.

Clock.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.	Screen.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.	Time of passing each screen. t.	Δ_1.	Δ_2.
″ 1	27·54	0	27·540	+22·065	−90	1	91·67	0	91·670	+2·173	+54	″ 0·0000	+995	+24
2	49·61	−·005	49·605	21·975	85	2	93·83	+·013	93·843	2·227	55	·0995	1019	26
3	71·58	0	71·580	21·890	−80	3	96·07	0	96·070	2·282	55	·2014	1045	26
4	93·47	0	93·470	+21·810		4	98·35	+·002	98·352	2·337	54	·3059	1071	25
5	115·28	0	115·280			5	100·69	−·001	100·689	2·391	55	·4130	1096	25
						6	103·08	0	103·080	2·446	54	·5226	1121	26
						7	105·53	−·004	105·526	2·500	53	·6347	1147	24
						8	108·04	−·014	108·026	2·553	+52	·7494	1171	+27
						9	110·59	−·011	110·579	+2·605		·8665	+1198	
						10	113·18	+·004	113·184			·9863		

Round 32†. Weight of shot 21·81 lbs.

Clock.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.	Screen.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.	Time of passing each screen. t.	Δ_1.	Δ_2.
″ 3	44·45	0	44·450	+21·630	−27	1	65·19	0	65·190	+2·239	+54	″ 0·0000	+1036	+25
4	66·08	0	66·080	21·603	−26	2	**	*	67·429	2·293	54	·1036	1061	25
5	87·68	+·003	87·683	+21·577		3	69·72	+·002	69·722	2·347	49	·2097	1086	23
6	109·26	0	109·260			4	72·07	−·001	72·069	2·396	50	·3183	1109	23
						5	74·46	+·005	74·465	2·446	52	·4292	1132	25
						6	76·91	+·001	76·911	2·498	57	·5424	1157	26
						7	79·41	−·001	79·409	2·555	59	·6581	1183	27
						8	81·97	−·006	81·964	2·614	+58	·7764	1210	+27
						9	84·58	−·002	84·578	+2·672		·8974	+1237	
						10	87·25	0	87·250			1·0211		

Summary (5). Ogival-headed Shot (one diameter), Hollow.

Time occupied by shot in passing from the first to each succeeding screen.

No. of round.	Screen 1.	Screen 2.	Screen 3.	Screen 4.	Screen 5.	Screen 6.	Screen 7.	Screen 8.	Screen 9.	Screen 10.
14	″ 0·0000	″ 0·1001	″ 0·2027	″ 0·3078	″ 0·4155	″ 0·5258	″ 0·6385	″ 0·7537	″ 0·8713	″ 0·9911
16	0·0000	·0986	·1994	·3026	·4084	·5168	·6278	·7414	·8576	·9764
18	0·0000	·0993	·2011	·3055	·4124	·5218	·6338	·7483	·8653	·9848
20	0·0000	·0990	·2007	·3050	·4119	·5212	·6329	·7470	·8635	·9823
22	0 0000	·0990	·2002	·3038	·4098	·5183	·6292	·7425	·8584	·9768
24	0·0000	·0995	·2016	·3061	·4132	·5228	·6350	·7497	·8671	·9870
26	0·0000	·0997	·2021	·3069	·4142	·5239	·6362	·7508	·8677	·9870
28	0·0000	·0991	·2005	·3044	·4109	·5198	·6312	·7452	·8616	·9804
30	0·0000	·0995	·2014	·3059	·4130	·5226	·6347	·7494	·8665	·9863
32†	0·0000	·1036	·2097	·3183	·4292	·5424	·6581	·7764	·8974	1·0211

† Gun loaded all night. The initial velocity is therefore reduced.

Velocities at the following distances from the gun.

No. of round.	150 ft.	300 ft.	450 ft.	600 ft.	750 ft.	900 ft.	1050 ft.	1200 ft.	1350 ft.
	f.s.	*f.s.*	*f.s.*	*f.s.*	*f.s.*	*f.s.*	*f.s.*	*f.s.*	*f.s.*
14	1499	1462	1427	1393	1360	1331	1302	1276	1252
16	1521	1488	1453	1418	1384	1351	1320	1291	1263
18	1511	1473	1437	1403	1371	1339	1310	1282	1255
20	1515	1475	1438	1403	1372	1343	1315	1288	1263
22	1515	1482	1448	1415	1382	1353	1324	1294	1267
24	1508	1469	1435	1401	1369	1337	1308	1278	1251
26	1505	1465	1431	1398	1367	1336	1309	1283	1257
28	1514	1479	1444	1408	1377	1347	1316	1289	1263
30	1508	1472	1435	1401	1369	1338	1308	1281	1252
32	1448	1414	1381	1353	1325	1296	1268	1240	1213

No. of round.	Weight of shot.	Values of bl^2.	Difference from mean value.
	lbs.		
14	21·78	·00125	·00000
16	21·81	·00124	−·00001
18	21·81	·00127	+·00002
20	21·83	·00127	+·00002
22	21·81	·00119	−·00006
24	21·83	·00127	+·00002
26	21·81	·00125	·00000
28	21·83	·00123	−·00002
30	21·81	·00126	+·00001
32	21·81	·00123	−·00002
Means	21·81	·00125	·00002

(6) Ogival-headed Shot (two diameters), Hollow.

Round 15. Weight of shot 21·92 lbs.

Clock.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.	Screen.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.	Time of passing each screen. t.	Δ_1.	Δ_2.
″ 1	3·55	0	3·550	+22·450	−60	1	44·59	−·003	44·587	+2·221	+55	″ 0·0000	+993	+26
2	26·00	0	26·000	22·390	90	2	46·80	+·008	46·808	2·276	55	·0993	1019	24
3	48·39	0	48·390	22·300	−120	3	49·08	+·004	49·084	2·331	55	·2012	1043	26
4	70·69	0	70·690	+22·180		4	51·43	−·015	51·415	2·386	54	·3055	1069	24
5	92·87	0	92·870			5	53·79	+·011	53·801	2·440	53	·4124	1093	24
						6	56·25	−·009	56·241	2·493	51	·5217	1117	24
						7	58·74	−·006	58·734	2·544	+50	·6334	1141	+24
						8	61·27	+·008	61·278	+2·594		·7475	+1165	
						9	63·87	+·002	63·872			·8640		

Round 21. Weight of shot 21·89 lbs.

Clock.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.	Screen.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.	Time of passing each screen. t.	Δ_1.	Δ_2.
″ 1	4·24	0	4·240	+20·420	−69	1	59·66	−·005	59·655	+2·018	+47	″ 0·0000	+994	+26
2	24·66	0	24·660	20·351	65	2	61·66	+·013	61·673	2·065	48	·0994	1020	24
3	45·02	−·009	45·011	20·286	61	3	63·74	−·002	63·738	2·113	48	·2014	1044	23
4	65·29	+·007	65·297	20·225	−57	4	65·85	+·001	65·851	2·161	48	·3058	1067	24
5	85·53	−·008	85·522	+20·168		5	68·00	+·012	68·012	2·209	47	·4125	1091	24
6	105·69	0	105·690			6	70·23	−·009	70·221	2·256	46	·5216	1115	23
						7	62·48	−·003	72·477	2·302	46	·6331	1138	23
						8	74·78	−·001	74·779	2·348	+45	·7469	1161	+24
						9	77·13	−·003	77·127	+2·393		·8630	+1185	
						10	79·52	0	79·520			·9815		

Round 23. Weight of shot 21·94 lbs.

Clock.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.	Screen.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.	Time of passing each screen. t.	Δ_1.	Δ_2.
″ 1	21·66	0	21·660	+23·340	−50	1	92·55	0	92·550	+2·288	+53	″ 0·0000	+ 986	+23
2	45·00	0	45·000	23·290	50	2	94·84	−·002	94·838	2·341	53	·0986	1009	24
3	68·28	+·010	68·290	23·240	50	3	97·18	−·001	97·179	2·394	55	·1995	1033	23
4	91·53	0	91·530	23·190	−50	4	99·57	+·003	99·573	2·449	56	·3028	1056	24
5	114·73	−·010	114·720	+23·140		5	102·01	+·012	102·022	2·505	55	·4084	1080	24
6	137·85	+·010	137·860			6	104·53	−·003	104·527	2·560	56	·5164	1104	25
						7	107·11	−·023	107·087	2·616	+56	·6268	1129	+23
						8	109·71	−·007	109·703	+2·672		·7397	+1152	
						9	112·37	+·005	112·375			·8549		

Round 25. Weight of shot 21·97 lbs.

Clock.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.	Screen.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.	Time of passing each screen. t.	Δ_1.	Δ_2.
″ 1	28·14	0	28·140	+21·315	−102	1	97·88	−·002	97·878	+2·087	+47	″ 0·0000	+ 993	+22
2	49·45	+·005	49·455	21·213	101	2	99·95	+·015	99·965	2·134	47	·0993	1015	24
3	70·67	−·002	70·668	21·112	102	3	102·10	−·001	102·099	2·181	46	·2008	1039	22
4	91·78	0	91·780	21·010	−100	4	104·28	0	104·280	2·227	47	·3047	1061	23
5	112·78	+·010	112·790	+20·910		5	106·50	+·007	106·507	2·274	47	·4108	1084	22
6	133·70	0	133·700			6	108·78	+·001	108·781	2·321	47	·5192	1106	23
						7	111·11	−·008	111·102	2·368	48	·6298	1129	24
						8	113·46	+·010	113·470	2·416	+48	·7427	1153	+24
						9	115·88	+·006	115·886	+2·464		·8580	+1177	
						10	118·35	0	118·350			·9757		

Round 27. Weight of shot 21·97 lbs.

Clock.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.	Screen.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.	Time of passing each screen. t.	Δ_1.	Δ_2.
″ 1	1·40	0	1·400	+19·430	−40	1	45·61	0	45·610	+1·916	+46	″ 0·0000	+ 989	+25
2	20·84	−·010	20·830	19·390	40	2	47·52	+·006	47·526	1·962	46	·0989	1014	24
3	40·21	+·010	40·220	19·350	−40	3	49·46	+·028	49·488	2·008	44	·2003	1038	23
4	59·57	0	59·570	+19·310		4	51·50	−·004	51·496	2·052	43	·3041	1061	22
5	78·88	0	78·880			5	53·54	+·008	53·548	2·095	44	·4102	1083	23
						6	55·65	−·007	55·643	2·139	43	·5185	1106	23
						7	57·78	+·002	57·782	2·182	44	·6291	1129	23
						8	59·96	+·004	59·964	2·226	+44	·7420	1152	+23
						9	62·19	0	62·190	+2·270		·8572	+1175	
						10	64·46	0	64·460			·9747		

Round 29. Weight of shot 21·97 lbs.

Clock.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.	Screen.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.	Time of passing each screen. t.	Δ_1.	Δ_2.
″ 1	16·26	0	16·260	+23·270	−60	1	62·14	0	62·140	+2·291	+56	″ 0·0000	+ 989	+24
2	39·53	0	39·530	23·210	73	2	64·44	−·009	64·431	2·347	56	·0989	1013	25
3	62·73	+·010	62·740	23·137	−85	3	66·77	+·008	66·778	2·403	54	·2002	1038	23
4	85·88	−·003	85·877	+23·052		4	69·20	−·019	69·181	2·457	53	·3040	1061	24
5	108·93	−·001	108·929			5	71·64	−·002	71·638	2·510	53	·4101	1085	23
						6	74·15	−·002	74·148	2·563	52	·5186	1108	23
						7	76·81	−·099	76·711	2·615	51	·6294	1131	22
						8	79·32	+·006	79·326	2·666	+52	·7425	1153	+23
						9	* *	*	81·992	+2·718		·8578	+1176	
						10	84·71	0	84·710			·9754		

Round 31. Weight of shot 21·91 lbs.

Clock.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.	Screen.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.	Time of passing each screen. t.	Δ_1.	Δ_2.
″ 1	13·22	−·010	13·210	+19·260	−80	1	57·06	0	57·060	+1·897	+41	″ 0·0000	+993	+22
2	32·46	+·010	32·470	19·180	90	2	58·96	−·003	58·957	1·938	41	·0993	1015	21
3	51·66	−·010	51·650	19·090	−100	3	60·89	+·005	60·895	1·979	43	·2008	1036	24
4	70·73	+·010	70·740	+18·990		4	62·87	+·004	62·874	2·022	44	·3044	1060	24
5	89·73	0	89·730			5	64·89	+·006	64·896	2·066	46	·4104	1084	24
						6	66·95	+·012	66·962	2·112	46	·5188	1108	25
						7	69·07	+·004	69·074	2·158	46	·6296	1133	25
						8	71·24	−·008	71·232	2·204	+46	·7429	1158	+25
						9	73·44	−·004	73·436	+2·250		·8587	+1183	
						10	75·68	+·006	75·686			·9770		

Round 33†. Weight of shot 21·94 lbs.

Clock.	Reading.	Correction.	Corrected reading.	Δ_1.	Screen.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.	Time of passing each screen. t.	Δ_1.	Δ_2.
″ 3	45·67	0	45·670	+18·580	1	59·32	0	59·320	+1·893	+40	″ 0·0000	+1018	+22
4	64·25	0	64·250	+18·570	2	61·21	+·003	61·213	1·933	42	·1018	1040	23
5	82·82	0	82·820		3	63·14	+·006	63·146	1·975	44	·2058	1063	23
					4	65·13	−·009	65·121	2·019	46	·3121	1086	26
					5	67·14	0	67·140	2·065	45	·4207	1112	25
					6	69·21	−·005	69·205	2·110	45	·5319	1137	24
					7	71·33	−·015	71·315	2·155	+47	·6456	1161	+26
					8	73·47	0	73·470	+2·202		·7617	+1187	
					9	75·67	+·002	75·672			·8804		

Summary (6). Ogival-headed Shot (two diameters), Hollow.

Time occupied by shot in passing from the first to each succeeding screen.

No. of round.	Screen 1.	Screen 2.	Screen 3.	Screen 4.	Screen 5.	Screen 6.	Screen 7.	Screen 8.	Screen 9.	Screen 10.
15	″ 0·0000	″ 0·0993	″ 0·2012	″ 0·3055	″ 0·4124	″ 0·5217	″ 0·6334	″ 0·7475	″ 0·8640	″ **
21	0·0000	·0994	·2014	·3058	·4125	·5216	·6331	·7469	·8630	·9815
23	0·0000	·0986	·1995	·3028	·4084	·5164	·6268	·7397	·8549	**
25	0·0000	·0993	·2008	·3047	·4108	·5192	·6298	·7427	·8580	·9757
27	0·0000	·0989	·2003	·3041	·4102	·5185	·6291	·7420	·8572	·9747
29	0·0000	·0989	·2002	·3040	·4101	·5186	·6294	·7425	·8578	·9754
31	0·0000	·0993	·2008	·3044	.4104	·5188	·6296	·7429	·8587	·9770
33	0·0000	·1018	·2058	·3121	·4207	·5319	·6456	·7617	·8804	**

Velocities at the following distances from the gun.

No. of round.	150 ft.	300 ft.	450 ft.	600 ft.	750 ft.	900 ft.	1050 ft.	1200 ft.	1350 ft.
	f.s.	*f.s.*	*f.s.*	*f.s.*	*f.s.*	*f.s.*	*f.s.*	*f.s.*	*f.s.*
15	1511	1472	1438	1403	1372	1343	1315	1288	**
21	1509	1471	1437	1406	1375	1345	1318	1292	1266
23	1521	1487	1452	1420	1389	1359	1329	1302	**
25	1511	1478	1444	1414	1384	1356	1329	1301	1274
27	1517	1479	1445	1414	1385	1356	1329	1302	1277
29	1517	1481	1445	1414	1382	1354	1326	1301	1276
31	1511	1478	1448	1415	1384	1354	1324	1295	1268
33	1473	1442	1411	1381	1349	1319	1292	1264	**

† Round 32 had remained in the gun all night. This probably accounts for the reduced initial velocity of Round 33.

No. of round.	Weight of shot.	Value of bl^2.	Difference from mean.
	lbs.		
15	21·92	·00124	+·00006
21	21·89	·00121	+·00003
23	21·94	·00118	·00000
25	21·97	·00114	−·00004
27	21·97	·00118	·00000
29	21·97	·00119	+·00001
31	21·91	·00116	−·00002
33	21·94	·00118	·00000
Means	21·94	·00118	·00002

Since the second differences of $0, t_2, t_3 \ldots\ldots t_{10}$ are nearly constant in the preceding experiments, we are led to the equation

$$t = as + bs^2, \text{ connecting space and time.}$$

This shows that if v be the velocity of the shot at time t, and f the retarding force, then

$$v = \frac{ds}{dt} = \frac{1}{a+2bs};$$

and if $v=\mathrm{V}$ when $s=0$, we have $\mathrm{V}=\frac{1}{a}$;

$$\therefore \quad v = \frac{1}{\frac{1}{\mathrm{V}}+2bs},$$

and

$$f = \frac{d^2s}{dt^2} = -2bv^3.$$

The values of bl^2 for each experiment were obtained as follows:—

$$\left.\begin{array}{llll} \text{Since} & & & t = as+bs^2 \\ \text{If} & s=l, & \text{then} & t_2 = al+bl^2, \\ & s=2l, & ,, & t_3 = 2al+4bl^2 \\ & \ldots & & \ldots \\ & s=\overline{n-1}l, & ,, & t_n = \overline{n-1}al+\overline{n-1}^2bl^2, \end{array}\right\}$$

or

$$\left.\begin{array}{r} \frac{t_2}{1} = al+\ bl^2, \\ \frac{t_3}{2} = al+2bl^2, \\ \ldots\ldots \\ \frac{t_n}{n-1} = al+\overline{n-1}bl^2. \end{array}\right\}$$

Finding, then, the numerical values of

$$\frac{t_2}{1}, \frac{t_3}{2}, \ldots \frac{t_n}{n-1}$$

for each experiment, and taking the difference of two of these quantities, we find

$$\frac{t_n}{n-1} - \frac{t_3}{2} = \overline{n-3}bl^2,$$

where $l=150$ feet, and the mean value of bl^2 so determined from each round may be taken to correspond to the mean velocity of that round.

Since the retarding force is $f=-2bv^3$, acting upon an experimental shot of weight W' lbs., the resistance of the air measured in pounds

$$=-2bv^3\frac{W'}{g};$$

and the resistance will always be the same against a shot of the same external form when moving with the velocity v. If W be the weight of a different shot, then the retarding force

$$f'=-2bv^3\frac{W'}{g}\times\frac{g}{W}=-2\left(b\frac{W'}{W}\right)v^3,$$

or b varies inversely as the weight of a shot of given external form. If we vary the diameter of the shot, it is usual to assume that the resistance of the air varies as the square of the diameter for similar forms of heads of shot. The values of $2b$ are given below in terms of R, the radius of the shot in feet, and W, its weight in pounds. But inasmuch as the diameters of shot are generally given in inches, the same values of $2b$ have been expressed in terms of d, the diameter of the shot in inches. Unfortunately there are only three successful rounds for each of the solid ogivals, and one value of bl^2 in each case is manifestly too large. This must have arisen from unsteadiness of the shot in its flight. If we reject these two, the values of bl^2 derived from the solid ogivals agree very well with those derived from the hollow shot of the like forms.

For the hemispherical head we have $bl^2=\cdot00084$ for shot of mean weight 39·34 lbs., and 4·7 inches in diameter. To find the value of $2b$ adapted for hemispherical-headed shot of weight W lbs., and diameter$=d$ inches$=2$R feet, we have

$$2b=\frac{2\times\cdot00084}{(150)^2}\times\left(\frac{d}{4\cdot7}\right)^2\left(\frac{39\cdot34}{W}\right)=\cdot0000001329\,\frac{d^2}{W},$$

or

$$=\frac{2\times\cdot00084}{150}\left(\frac{24R}{4\cdot7}\right)^2\frac{39\cdot34}{W}=\cdot000077\,\frac{R^2}{W},\text{ and so on for the rest.}$$

Table of values of $2b$ for differently formed heads adapted for elongated shot of weight W lbs. and radius R feet, or diameter d inches.

Form of head.	Experimental value of bl^2.	Mean weight of experimental shot.	Value of $2b$ when diameter $=2$R feet.	Value of $2b$ when diameter $=d$ inches.
		lbs.		
(1) Hemispherical	·00084	39·34	$\cdot000077\frac{R^2}{W}$	$\cdot0000001329\frac{d^2}{W}$
(2) Hemispheroidal...........	·00067	38·70	$\cdot000060\frac{R^2}{W}$	$\cdot0000001043\frac{d^2}{W}$
(3) Ogival (1 diameter)	·00070	39·56	$\cdot000064\frac{R^2}{W}$	$\cdot0000001114\frac{d^2}{W}$
(4) Ogival (2 diameters)......	·00070	38·52	$\cdot000062\frac{R^2}{W}$	$\cdot0000001085\frac{d^2}{W}$
(5) Ogival (1 diameter)	·00125	21·81	$\cdot000063\frac{R^2}{W}$	$\cdot0000001097\frac{d^2}{W}$
(6) Ogival (2 diameters) ...	·00118	21·94	$\cdot000060\frac{R^2}{W}$	$\cdot0000001042\frac{d^2}{W}$

Although the motion of a shot may be well represented by supposing a retarding force $=-2bv^3$ to act through a range of 1400 feet, there is reason to suppose that for velocities ranging from 1500 to 900 feet per second the value of b will be less for the lower velocities with equal degrees of steadiness. It unfortunately happens, however, that the angular velocity imparted to a shot, which most probably remains little changed during the time of flight, depends directly upon the initial velocity of the shot. Hence, when shot are fired with low initial velocities with a view to determine the value of b for low velocities, the steadiness of the shot is diminished, and therefore there is an increase of the resistance of the air on this account. The only way to meet the difficulty is to place screens near the gun to find the initial velocity, and others at a distance of 2000 yards or more, and so compare theory and experiment.

It is worthy of notice that if a body move in a straight line under the action of a force varying as the velocity cubed, the mean velocity obtained by dividing space by time is exactly the actual velocity at the middle point of that space. Thus

$$\frac{\text{space } 2s}{\text{time of describing space } 2s}=\frac{2s}{2as+4bs^2}=\frac{1}{a+2bs}=\text{velocity at distance } s.$$

The date of the Report of the above experiments was October 23, 1866.

I have long been aware that Major OTTO had made trial of various laws of the resistance of the air in a work published in 1855. The law of the cube of the velocity was tried, but without any definite result*. It was in April 1867 that I first learnt that M. HÉLIE† had proposed the law of the cube of the velocity as the law of the resistance of the air to elongated projectiles in a work dated 1865, which law he had deduced from experiments made at Gâvre in 1859, 1860, and 1861‡. It will be convenient to quote his own statement of the best series of experiments made at Gâvre in 1859, in order to show the nature of the work done, and the perfect independence of my own methods and numerical results. M. HÉLIE used one of the electro-ballistic pendulums to measure his velocities, but he does not state distinctly which it was.

"Si la résistance de l'air est réellement proportionnelle au cube de la vitesse, on doit avoir un résultat sensiblement constant en substituant, dans l'expression $\frac{v'-v''}{v'v''x}$, les valeurs de v', v'' et x correspondantes à la même charge.

* Hilfsmittel für ballistische Rechnungen, 1855, p. 12.

† Traité de Balistique, 1865.

‡ [In a Memoir, "Études de Balistique expérimentale," presented to the Belgian Academy by Captain P. C. BOULENGÉ, June 12, 1867, the author, having deduced the cubic law of resistance of the air from his experiments, proceeds to remark:—

"Ce résultat est en accord complet avec les travaux les plus récents faits en France; en effet, les expériences exécutées par la commission des principes du tir, en 1856 et 1857, ont conduit M. le capitaine WELTER, professeur à l'École d'application de l'artillerie et du génie de Metz, à reconnaître que la résistance de l'air sur les projectiles sphériques est simplement proportionnelle au cube de la vitesse.

"Cette loi, admise depuis 1862, comme base des études balistiques à cette école, a fourni des formules très-simples et très-facilement calculables sans l'intervention de tables, se prêtant à des recherches que les anciennes formules balistiques ne permettaient pas d'aborder, et donnant des résultats plus conformes à la pratique" (p. 84). —Aug. 1, 1868.]

"Résultats moyens des expériences.
(Chaque vitesse est déduite de 30 coups.)

Charge du canon.	Première vitesse, v', à 33 mètres du canon.	Dernière vitesse, v''.	Intervalle des points d'observation, x.	Valeur de $\frac{v'-v''}{v'v''x}$ ou c.
kilog.	mètr.	mètr.	mètr.	
1·5	225·1	215·6	464	0·000000422
2·0	263·7	252·5	467	0·000000360
2·5	291·9	275·9	467	0·000000425
3·0	309·6	291·8	467	0·000000422
3·5	326·9	306·4	467	0·000000438

"Sauf l'anomalie que présente la charge de $2^{k}\cdot0$ la valeur de $\frac{v'-v''}{v'v''x}$ se montre sensiblement constante. La formule $r=cv^3$ est donc suffisamment justifiée"*. The diameter of the shot was $0^{m}\cdot1623=6^{ins}\cdot39$, its weight $30^{k}=66^{lbs}\cdot14$, and its form hollow ogival with head struck with a radius of nearly two diameters.

In order to facilitate the comparison of this experiment with my own, I have converted the French into English measures.

Charge.	v'.	v''.	x.	$c=2b$.
lbs.	*f.s.*	*f.s.*	ft.	
3·31	738·5	707·4	1522·4	0·0000000391
4·41	865·1	828·4	1532·2	0·0000000334
5·51	957·7	905·2	1532·2	0·0000000395
6·61	1015·8	957·4	1532·2	0·0000000392
7·72	1072·5	1005·3	1532·2	0·0000000407
			Mean value of c or $2b=$	0·0000000384
				$=\cdot0000358\frac{R^2}{W}$ if R ft. = radius of shot.
				$=\cdot000000069\frac{d^2}{W}$ if d in. = diameter of shot.

The values of $2b$ given by my own experiments for shot of the same form were

$$\cdot000060\frac{R^2}{W} \text{ or } \cdot0000001042\frac{d^2}{W}$$

for velocities ranging from 1520 to 1270 *f.s.*; so that the value of $2b$ deduced from my own experiments might be expected to be sensibly greater than its value deduced from M. HÉLIE's experiments made with lower velocities. There are three or four other tabular statements of less complete experiments given by M. HÉLIE. In the present state of the question it is impossible to make any more exact comparison of the two systems of experimenting†.

* HÉLIE, Traité, pp. 407, 408.

† [An extended series of experiments just completed at Shoeburyness, with 3, 5, 7, and 9-inch elongated shot, has shown conclusively that, although the motion of a shot may be well represented by supposing the resistance of the air to vary as the cube of the velocity, and to be equal to $-2bv^3$ for a range of 1200 or 1300 feet, still

Meteorological Register, Shoeburyness.

Date.	Hour.		Barometer.		Thermometer.		Wind.		Weather and Remarks.
	A.M.	P.M.	Reading.	Temperature.	Dry.	Wet.	Direction.	Force.	
1866.									
Sept. 25th	10	...	30·05	61	61	54	S.W. by S.	2	Clear.
	...	3	30·05	61	61	55	S.W. by W.	2	Clear.
Sept. 26th	10	...	30·05	61	61	59	S.S.W.	3	Hazy.
	...	3	30·00	61	60	59	S.W. by S.	3	Ditto.
Sept. 27th	10	...	29·95	59	56	56	N.W. by W.	2	Ditto with rain.
	...	3	29·95	60	59	58	W.	2	Hazy.

Direction of firing from the North to South.

---

b is subject to considerable variations for large variations in v. For ogival heads struck with a radius of $1\frac{1}{2}$ diameter of the shot, fired with great steadiness, the following values of v and b have been found.

v.	$2b\frac{W}{d^2}$.
f.s.	
900	0·0000000604
950	0·0000000640
1000	0·0000000720
1050	0·0000000868
1100	0·0000001005
1200	0·0000001085
1300	0·0000001046
1400	0·0000000977
1500	0·0000000956
1600	0·0000000925

It thus appears that M. HÉLIE's value of $2b$ or c is true only for velocities about 950 *f.s.*—Aug. 1, 1868.]